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Massive superstring vertex operator in $D = 10$ superspace

Nathan Berkovits
Instituto de Física Teórica, Universidade Estadual Paulista
Rua Pamplona, 145, 01405-900, São Paulo, SP, Brasil
E-mail: nberkovi@ift.unesp.br

Osvaldo Chandía
Facultad de Física, Pontificia Universidad Católica de Chile
Casilla 306, Santiago 22, Chile
E-mail: ochandia@maxwell.fis.puc.cl

Abstract: Using the pure spinor formalism for the superstring, the vertex operator for the first massive states of the open superstring is constructed in a manifestly super-Poincaré covariant manner. This vertex operator describes a massive spin-two multiplet in terms of ten-dimensional superfields.

Keywords: Superstrings and Heterotic Strings, Superspaces.

Contents

1. Introduction

To study the implications of spacetime supersymmetry for the superstring, it is useful to have a formalism in which super-Poincaré covariance is manifest. Although the super-Poincaré covariant Green-Schwarz formalism [1] can be used to classically describe the superstring, it has not yet been quantized in a covariant manner. This prevented the construction of super-Poincaré covariant expressions for massive vertex operators since, unlike massless vertex operators, massive vertex operators cannot be obtained from the classical action in a curved background.

Recently, a new super-Poincaré covariant formalism for the superstring has been proposed using a BRST-like operator $Q = \int \lambda^\alpha d_\alpha$, where d_α is the $D = 10$ supersymmetric derivative and λ^α is a pure spinor variable [2]. In this formalism, physical vertex operators are defined in a manifestly super-Poincaré covariant manner as states in the cohomology of Q. Massless vertex operators have been explicitly constructed using this formalism and tree amplitudes have been shown to coincide with Ramond-Neveu-Schwarz amplitudes [3, 4].

In this paper, the vertex operator for the first massive states of the open superstring will be explicitly constructed in super-Poincaré covariant notation and shown to describe a massive spin-two multiplet containing 128 bosonic and 128 fermionic degrees of freedom. Although this construction is guaranteed to succeed because of the cohomology arguments of [5], it is interesting to see how the vertex operator for this massive multiplet is expressed in terms of ten-dimensional superfields.

2. Physical vertex operator

Physical states in the pure spinor formalism for the open superstring are defined as ghost-number one states in the cohomology of $Q = \int \lambda^\alpha d_\alpha$, where λ^α is a pure spinor variable constrained to satisfy $\lambda\gamma^m\lambda = 0$,

$$d_\alpha = p_\alpha - \frac{1}{2}\gamma^m_{\alpha\beta}\theta^\beta\partial x_m - \frac{1}{8}\gamma^m_{\alpha\beta}\gamma_{m\gamma\delta}\theta^\beta\theta^\gamma\partial\theta^\delta\,, \tag{2.1}$$

$\gamma_m^{\alpha\beta}$ and $\gamma_{m\alpha\beta}$ are 16×16 symmetric matrices which are the off-diagonal components of the 32×32 ten-dimensional gamma matrices, $[x^m, \theta^\alpha, p_\alpha]$ for $m = 0$ to 9 and $\alpha = 1$ to 16 are free worldsheet fields satisfying the OPE's

$$x^m(y)x^n(z) \longrightarrow -\alpha'\eta^{mn}(\log|y-z| + \log|y-\bar{z}|)\,, \qquad p_\alpha(y)\theta^\beta(z) \longrightarrow \frac{\alpha'}{2(y-z)}\delta^\beta_\alpha\,, \tag{2.2}$$

and α' is the inverse of the string tension. One can use (2.2) to show that d_α is spacetime supersymmetric and satisfies the OPE's [6]

$$\begin{aligned} d_\alpha(y)d_\beta(z) &\longrightarrow -\frac{\alpha'}{2(y-z)}\gamma^m_{\alpha\beta}\Pi_m(z)\,, \qquad d_\alpha(y)\Pi^m(z) \longrightarrow \frac{\alpha'}{2(y-z)}\gamma^m_{\alpha\beta}\partial\theta^\beta(z)\,,\\ \Pi^m(y)V(z) &\longrightarrow -\frac{\alpha'}{y-z}\partial^m V(z)\,, \qquad d_\alpha(y)V(z) \longrightarrow \frac{\alpha'}{2(y-z)}D_\alpha V(z)\,, \end{aligned} \tag{2.3}$$

where $\Pi^m = \partial x^m + \frac{1}{2}\gamma^m_{\alpha\beta}\theta^\alpha\partial\theta^\beta$, $V(x,\theta)$ is an arbitrary ten-dimensional superfield, and $D_\alpha = \frac{\partial}{\partial\theta^\alpha} + \gamma^m_{\alpha\beta}\theta^\beta\partial_m$ is the supersymmetric covariant derivative which satisfies $\{D_\alpha, D_\beta\} = 2\gamma^m_{\alpha\beta}\partial_m$.

The pure spinor constraint $\lambda\gamma^m\lambda = 0$ implies that the canonical momentum for λ^α, which will be called w_α, only appears in combinations which are invariant under the gauge transformation $\delta w_\alpha = (\gamma^m\lambda)_\alpha\Lambda_m$ for arbitrary Λ_m. This implies that w_α only appears in the Lorentz-covariant combinations $N_{mn} = \frac{1}{2}(w\gamma_{mn}\lambda)$ and $J = w_\alpha\lambda^\alpha$. By solving $\lambda\gamma^m\lambda = 0$ in terms of unconstrained fields, one can show that N_{mn} and J satisfy the OPE's [4]

$$\begin{aligned} N^{mn}(y)\lambda^\alpha(z) &\longrightarrow \frac{\alpha'}{4(y-z)}(\gamma^{mn})^\alpha{}_\beta\lambda^\beta(z), \qquad J(y)\lambda^\alpha(z) \longrightarrow \frac{\alpha'}{2(y-z)}\lambda^\alpha(z)\,,\\ N^{kl}(y)N^{mn}(z) &\longrightarrow -\frac{3(\alpha')^2}{4(y-z)^2}\eta^{k[n}\eta^{m]l} + \frac{\alpha'}{2(y-z)}\left(\eta^{m[l}N^{k]n}(z) - \eta^{m[l}N^{k]m}(z)\right)\,,\\ J(y)J(z) &\longrightarrow -\frac{(\alpha')^2}{(y-z)^2}\,. \end{aligned} \tag{2.4}$$

Furthermore, $\lambda\gamma^m\lambda = 0$ implies that N^{mn} and J satisfy the relation

$$:N^{mn}\lambda^\alpha:\gamma_{m\alpha\beta} - \frac{1}{2}:J\lambda^\alpha:\gamma^n_{\alpha\beta} = \alpha'\gamma^n_{\alpha\beta}\partial\lambda^\alpha(z)\,, \tag{2.5}$$

where the normal-ordered product is defined as

$$:U^A(z)\lambda^\alpha(z) := \oint \frac{dy}{y-z}U^A(y)\lambda^\alpha(z)\,.$$

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To prove (2.5), note that w_α drops out of the left-hand side because $\lambda\gamma^m\lambda = 0$. And the coefficient in the normal ordering contribution $\alpha'\gamma^n_{\alpha\beta}\partial\lambda^\alpha$ can be determined by computing the double pole of (2.5) with J using the OPE $J(y)J(z) \to -(\alpha')^2/(y-z)^2$.

When $\alpha'(\text{mass})^2 = n$, open superstring vertex operators are constructed from arbitrary combinations of $[x^m, \theta^\alpha, d_\alpha, \lambda^\alpha, N^{mn}, J]$ which carry ghost number one and conformal weight n at zero momentum. Note that $[d_\alpha, N_{mn}, J]$ carry conformal weight one and λ^α carries ghost number one. For example, the most general vertex operator at $(\text{mass})^2 = 0$ is $V = \lambda^\alpha A_\alpha(x,\theta)$, where $A_\alpha(x,\theta)$ is an unconstrained spinor superfield [2, 7, 8]. One can easily check that $QV = 0$ and $\delta V = Q\Omega$ implies $\gamma^{\alpha\beta}_{mnpqr} D_\alpha A_\beta = 0$ and $\delta A_\alpha = \frac{\alpha'}{2} D_\alpha \Omega$, which are the super-Maxwell equations of motion and gauge invariances written in terms of a spinor superfield.

When $\alpha'(\text{mass})^2 = 1$, the first massive states of the open superstring are described by the vertex operator

$$\begin{aligned} V &= \partial\lambda^\alpha A_\alpha(x,\theta) + :\partial\theta^\beta\lambda^\alpha B_{\alpha\beta}(x,\theta): + :d_\beta\lambda^\alpha C^\beta{}_\alpha(x,\theta): + \\ &+ :\Pi^m\lambda^\alpha H_{m\alpha}(x,\theta): + :J\lambda^\alpha E_\alpha(x,\theta): + :N^{mn}\lambda^\alpha F_{\alpha mn}(x,\theta):, \end{aligned} \tag{2.6}$$

where $:U^A\lambda^\alpha\Phi_{\alpha A}(x,\theta)(z) := \oint \frac{dy}{y-z} U^A(y)\lambda^\alpha(z)\Phi_{\alpha A}(z)$ and $\Phi_{\alpha A}(x,\theta)$ are the various superfields appearing in (2.6). Note that because of (2.5), V is invariant under the field redefinition

$$\delta F_{\alpha mn} = \gamma_{m\alpha\beta}\Lambda^\beta_n - \gamma_{n\alpha\beta}\Lambda^\beta_m\,, \qquad \delta E_\alpha = -\gamma^m_{\alpha\beta}\Lambda^\beta_m\,, \qquad \delta A_\alpha = -2\alpha'\gamma^m_{\alpha\beta}\Lambda^\beta_m\,. \tag{2.7}$$

As will now be shown, the equations of motion and gauge invariances implied by $QV = 0$ and $\delta V = Q\Omega$ imply that the superfields $\Phi_{\alpha A}(x,\theta)$ describe a massive spin-two multiplet containing 128 bosonic and 128 fermionic degrees of freedom.

3. Equations of motion

Using the OPE's of (2.3) and (2.4), one finds that

$$\begin{aligned} \frac{2}{\alpha'}QV = \oint \frac{dy}{y-z}\Big[&\partial\lambda^\beta(y)\lambda^\alpha(z)(D_\alpha A_\beta(z) + B_{\alpha\beta}(z)) - \partial\theta^\gamma(y)\lambda^\alpha(z)\lambda^\beta(z)D_\beta B_{\alpha\gamma}(z) - \\ &- \gamma_{m\gamma\beta}\lambda^\gamma(y)\Pi^m(y)\lambda^\alpha(z)C^\beta{}_\alpha(z) - d_\gamma(y)\lambda^\alpha(z)\lambda^\beta(z)D_\beta C^\gamma{}_\alpha(z) + \\ &+ \gamma^m_{\gamma\delta}\lambda^\gamma(y)\partial\theta^\delta(y)\lambda^\alpha(z)H_{m\alpha}(z) + \Pi^m(y)\lambda^\alpha(z)\lambda^\beta(z)D_\beta H_{m\alpha}(z) - \\ &- \lambda^\gamma(y)d_\gamma(y)\lambda^\alpha(z)E_\alpha(z) + J(y)\lambda^\alpha(z)\lambda^\beta(z)D_\beta E_\alpha(z) - \\ &- \frac{1}{2}(\gamma^{mn})^\gamma{}_\delta\lambda^\delta(y)d_\gamma(y)\lambda^\alpha(z)F_{\alpha mn}(z) + N^{mn}(y)\lambda^\alpha(z)\lambda^\beta(z)D_\beta F_{\alpha mn}(z)\Big] \end{aligned} \tag{3.1}$$

$$\begin{aligned} &= -:\partial\theta^\gamma\lambda^\alpha\lambda^\beta\left[D_\alpha B_{\beta\gamma} - \gamma^m_{\alpha\gamma}H_{m\beta}\right]: + :\Pi^m\lambda^\alpha\lambda^\beta\left[D_\alpha H_{m\beta} - \gamma_{m\alpha\gamma}C^\gamma{}_\beta\right]: - \\ &- :d_\gamma\lambda^\alpha\lambda^\beta\left[D_\alpha C^\gamma{}_\beta + \delta^\gamma_\alpha E_\beta + \frac{1}{2}(\gamma^{mn})^\gamma{}_\alpha F_{\beta mn}\right]: + \\ &+ \lambda^\alpha\partial\lambda^\beta\left[D_\alpha A_\beta + B_{\alpha\beta} + \alpha'\gamma^m_{\beta\gamma}\partial_m C^\gamma{}_\alpha - \frac{\alpha'}{2}D_\beta E_\alpha + \frac{\alpha'}{4}(\gamma^{mn}D)_\beta F_{\alpha mn}\right] + \\ &+ :J\lambda^\alpha\lambda^\beta D_\alpha E_\beta: + :N^{mn}\lambda^\alpha\lambda^\beta D_\alpha F_{\beta mn}:, \end{aligned} \tag{3.2}$$

where $:U^A\lambda^\alpha\lambda^\beta\Sigma_{\alpha\beta A}(x,\theta)(z) := \oint \frac{dy}{y-z}U^A(y)\lambda^\alpha(z)\lambda^\beta(z)\Sigma_{\alpha\beta A}(z)$.

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Since $\lambda\gamma^m\lambda = \lambda\gamma^m\partial\lambda = 0$, $Qv = 0$ implies that the superfields $\Phi_{\alpha A}$ satisfy

$$
\begin{aligned}
&(\gamma_{mnpqr})^{\alpha\beta}[D_\alpha B_{\beta\gamma} - \gamma^s_{\alpha\gamma} H_{s\beta}] = 0\,,\\
&(\gamma_{mnpqr})^{\alpha\beta}[D_\alpha H_{s\beta} - \gamma_{s\alpha\gamma} C^\gamma{}_\beta] = 0\,,\\
&(\gamma_{mnpqr})^{\alpha\beta}\left[D_\alpha C^\gamma{}_\beta + \delta^\gamma_\alpha E_\beta + \frac{1}{2}(\gamma^{st})^\gamma{}_\alpha F_{\beta st}\right] = 0\,,\\
&(\gamma_{mnpqr})^{\alpha\beta}\left[D_\alpha A_\beta + B_{\alpha\beta} + \alpha'\gamma^s_{\beta\gamma}\partial_s C^\gamma{}_\alpha - \frac{\alpha'}{2} D_\beta E_\alpha + \frac{\alpha'}{4}(\gamma^{st}D)_\beta F_{\alpha st}\right] =\\
&\qquad = 2\alpha'\gamma^{\alpha\beta}_{mnpqr}\gamma^{vwxys}_{\alpha\beta}\eta_{st}K^t_{vwxy}\,,\\
&(\gamma_{mnp})^{\alpha\beta}\left[D_\alpha A_\beta + B_{\alpha\beta} + \alpha'\gamma^s_{\beta\gamma}\partial_s C^\gamma{}_\alpha - \frac{\alpha'}{2} D_\beta E_\alpha + \frac{\alpha'}{4}(\gamma^{st}D)_\beta F_{\alpha st}\right] =\\
&\qquad = 16\alpha'\gamma^{\alpha\beta}_{mnp}\gamma^{wxy}_{\alpha\beta}K^s_{wxys}\,,\\
&\gamma^{\alpha\beta}_{mnpqr} D_\alpha E_\beta = \gamma^{\alpha\beta}_{mnpqr}\,(\gamma^{vwxy}\gamma_s)_{\alpha\beta}\,K^s_{vwxy}\,,\\
&\gamma^{\alpha\beta}_{mnpqr} D_\alpha F^{st}_\beta = -\gamma^{\alpha\beta}_{mnpqr}\left(\gamma^{vwxy}\gamma^{[s}\right)_{\alpha\beta} K^{t]}_{vwxy}\,,
\end{aligned}
\tag{3.3}
$$

where K^s_{vwxy} is an arbitrary superfield. The possibility of introducing K^t_{vwxy} into the right-hand side of (3.3) comes from the fact that for arbitrary K^s_{vwxy},

$$
\begin{aligned}
&- :N_{st}\lambda^\alpha\lambda^\beta: \left(\gamma^{vwxy}\gamma^{[s}\right)_{\alpha\beta} K^{t]}_{vwxy} + :J\lambda^\alpha\lambda^\beta: (\gamma^{vwxy}\gamma_s)_{\alpha\beta} K^s_{vwxy} +\\
&+ \alpha'\lambda^\alpha\partial\lambda^\beta\left[2\gamma^{vwxys}_{\alpha\beta}\eta_{st}K^t_{vwxy} + 16\gamma^{wxy}_{\alpha\beta}K^s_{wxys}\right] = 0\,,
\end{aligned}
\tag{3.4}
$$

which follows from the identity

$$
:N_{st}\lambda^\alpha\lambda^\beta: \gamma^s_{\beta\gamma} - \frac{1}{2} :J\lambda^\alpha\lambda^\beta: \gamma_{t\beta\gamma} = \frac{5\alpha'}{4}\lambda^\alpha\partial\lambda^\beta\gamma_{t\beta\gamma} - \frac{\alpha'}{4}\lambda^\delta\partial\lambda^\beta(\gamma_{st})^\alpha{}_\delta\gamma^s_{\beta\gamma}\,. \tag{3.5}
$$

To derive (3.5), first define

$$
:U^A\lambda^\alpha\lambda^\beta: := \lim_{w\to z}\oint_{C_w}\frac{dy}{y-z}U^A(y)\lambda^\alpha(w)\lambda^\beta(z) + \lim_{w\to z}\oint_{C_z}\frac{dy}{y-z}U^A(y)\lambda^\alpha(w)\lambda^\beta(z)\,, \tag{3.6}
$$

where C_w encircles the point w and C_z encircles the point z. Using (2.4) and (2.5), one finds

$$
\begin{aligned}
&:N_{st}\lambda^\alpha\lambda^\beta: \gamma^s_{\beta\gamma} - \frac{1}{2} :J\lambda^\alpha\lambda^\beta: \gamma_{t\beta\gamma} =\\
&\quad = \alpha'\lambda^\alpha\partial\lambda^\beta\gamma_{t\beta\gamma} + \lim_{w\to z}\oint_{C_w}\frac{dy}{y-z}\left(N_{st}(y)\gamma^s_{\beta\gamma} - \frac{1}{2}J(y)\gamma_{t\beta\gamma}\right)\lambda^\alpha(w)\lambda^\beta(z)\\
&\quad = \alpha'\lambda^\alpha\partial\lambda^\beta\gamma_{t\beta\gamma} + \lim_{w\to z}\frac{1}{w-z}\left[\frac{\alpha'}{4}(\gamma_{st})^\alpha{}_\delta\lambda^\delta(w)\gamma^s_{\beta\gamma} - \frac{\alpha'}{4}\lambda^\alpha(w)\gamma_{t\beta\gamma}\right]\lambda^\beta(z)\\
&\quad = \alpha'\lambda^\alpha\partial\lambda^\beta\gamma_{t\beta\gamma} - \frac{\alpha'}{4}\lambda^\delta\partial\lambda^\beta(\gamma_{st})^\alpha{}_\delta\gamma^s_{\beta\gamma} + \frac{\alpha'}{4}\lambda^\alpha\partial\lambda^\beta\gamma_{t\beta\gamma}\,.
\end{aligned}
\tag{3.7}
$$

4. Gauge transformations

In order to determine the physical content of the equations of motion (3.3), one needs to gauge fix the superfields using the gauge transformations implied by $\delta V = Q\Omega$, as well as the transformations implied by the field redefinition of (2.7). Since the gauge parameter Ω should have ghost number zero and conformal weight one, the most general gauge parameter is

$$\Omega = :\partial\theta^\alpha\Omega_{1\alpha}(x,\theta): + :d_\alpha\Omega_2^\alpha(x,\theta): + :\Pi^m\Omega_{3m}(x,\theta): + \\ + :J\Omega_4(x,\theta): + :N^{mn}\Omega_{5mn}(x,\theta):, \tag{4.1}$$

where $:U^A\Omega_A(x,\theta) := \oint \frac{dy}{y-z}U^A(y)\ \Omega_A(z)$.

Using the OPE's of (2.3) and (2.4), one finds that

$$\frac{2}{\alpha'}Q\Omega = \oint \frac{dy}{y-z}\Big[\partial\lambda^\alpha(y)\Omega_{1\alpha}(z) - \partial\theta^\beta(y)\lambda^\alpha(z)D_\alpha\Omega_{1\beta}(z) - \gamma_{m\alpha\beta}\lambda^\beta(y)\Pi^m(y)\Omega_2^\alpha(z) - \\ - d_\beta(y)\lambda^\alpha(z)D_\alpha\Omega_2^\beta(z) + \gamma^m_{\gamma\delta}\lambda^\gamma(y)\partial\theta^\delta(y)\Omega_{3m}(z) + \\ + \Pi^m(y)\lambda^\alpha(z)D_\alpha\Omega_{3m}(z) - \lambda^\gamma(y)d_\gamma(y)\Omega_4(z) + J(y)\lambda^\alpha(z)D_\alpha\Omega_4(z) - \\ - \frac{1}{2}(\gamma^{mn})^\gamma{}_\delta\lambda^\delta(y)d_\gamma(y)\Omega_{5mn}(z) + N^{mn}(y)\lambda^\alpha(z)D_\alpha\Omega_{5mn}(z)\Big] \tag{4.2}$$

$$= \partial\lambda^\alpha\left[\Omega_{1\alpha} + \alpha'\gamma^m_{\alpha\beta}\partial_m\Omega_2^\beta - \frac{\alpha'}{2}D_\alpha\Omega_4 - \frac{\alpha'}{4}(\gamma^{mn})^\beta{}_\alpha D_\beta\Omega_{5mn}\right] + \\ + :\partial\theta^\beta\lambda^\alpha\left[-D_\alpha\Omega_{1\beta} + \gamma^m_{\alpha\beta}\Omega_{3m}\right]: + :d_\beta\lambda^\alpha\left[-D_\alpha\Omega_2^\beta - \delta_\alpha^\beta\Omega_4 - \frac{1}{2}(\gamma^{mn})^\beta{}_\alpha\Omega_{5mn}\right]: + \\ + :\Pi^m\lambda^\alpha\left[D_\alpha\Omega_{3m} - \gamma_{m\alpha\beta}\Omega_2^\beta\right]: + :J\lambda^\alpha D_\alpha\Omega_4: + :N^{mn}\lambda^\alpha D_\alpha\Omega_{5mn}: . \tag{4.3}$$

So $\delta V = \frac{2}{\alpha'}Q\Omega$ implies the following gauge transformations for the superfields in (2.6):

$$\begin{aligned} \delta A_\alpha &= \Omega_{1\alpha} + \alpha'\gamma^m_{\alpha\beta}\partial_m\Omega_2^\beta - \frac{\alpha'}{2}D_\alpha\Omega_4 - \frac{\alpha'}{4}(\gamma^{mn})^\beta{}_\alpha D_\beta\Omega_{5mn}\,, \\ \delta B_{\alpha\beta} &= -D_\alpha\Omega_{1\beta} + \gamma^m_{\alpha\beta}\Omega_{3m}\,, \\ \delta C^\beta{}_\alpha &= -D_\alpha\Omega_2^\beta - \delta_\alpha^\beta\Omega_4 - \frac{1}{2}(\gamma^{mn})^\beta{}_\alpha\Omega_{5mn}\,, \\ \delta H_{m\alpha} &= D_\alpha\Omega_{3m} - \gamma_{m\alpha\beta}\Omega_2^\beta\,, \\ \delta E_\alpha &= D_\alpha\Omega_4\,, \\ \delta F_{\alpha mn} &= D_\alpha\Omega_{5mn}\,. \end{aligned} \tag{4.4}$$

5. Massive spin-two multiplet

In this section, we shall show that the equations of motion of (3.3) and the gauge transformations of (4.4) and (2.7) imply that the superfields appearing in (2.6) describe a spin-two multiplet with $(\text{mass})^2 = 1/\alpha'$. Note that the 128 bosonic and 128 fermionic component fields in a massive spin-two multiplet consist of a traceless symmetric tensor g_{mn}, a three-form b_{mnp}, and a spin-3/2 field $\psi_{m\alpha}$ satisfying the equations:

$$\eta^{mn}g_{mn} = \partial^m g_{mn} = \partial^m b_{mnp} = \partial^m\psi_{m\alpha} = \gamma^{m\alpha\beta}\psi_{m\beta} = 0\,. \tag{5.1}$$

These ten-dimensional component fields can be understood as Kaluza-Klein modes of an eleven-dimensional supergravity multiplet.

The first equation of motion of (3.3) implies that $\lambda^\alpha\lambda^\beta\lambda^\gamma D_\alpha B_{\beta\gamma} = 0$, where $\lambda^\beta\lambda^\gamma B_{\beta\gamma} = (\lambda\gamma^{mnpqr}\lambda)B_{mnpqr}$. As discussed in ref. [9], this is the same equation of motion as for the super-Maxwell antifield $A^*_{\beta\gamma} = A^*_{mnpqr}(\gamma^{mnpqr})_{\beta\gamma}$. But up to the gauge transformation $\delta A^*_{mnpqr} = \gamma^{\beta\gamma}_{mnpqr} D_\beta \Lambda_\gamma$, $\lambda^\alpha\lambda^\beta\lambda^\gamma D_\alpha A^*_{\beta\gamma} = 0$ has only massless solutions. Since the fourth equation of (3.3) implies at $\alpha' = 0$ that $\gamma^{\alpha\beta}_{mnpqr}(B_{\alpha\beta} + D_\alpha A_\beta) = 0$, B_{mnpqr} has no massless solutions. So $\lambda^\alpha\lambda^\beta\lambda^\gamma D_\alpha B_{\beta\gamma} = 0$ implies that $B_{mnpqr} = \gamma^{\beta\gamma}_{mnpqr} D_\beta \Lambda_\gamma$ for some Λ_γ. Using the gauge parameters $\Omega_{1\alpha}$ and Ω_{3m}, one can therefore gauge $B_{\alpha\beta} = \gamma^{mnp}_{\alpha\beta} B_{mnp}$. Note that this gauge-fixing condition still leaves gauge invariances parameterized by $\Omega_{1\alpha}$ that satisfy $\gamma^{\alpha\beta}_{mnpqr} D_\alpha \Omega_{1\beta} = 0$.

Plugging $B_{\alpha\beta} = \gamma^{mnp}_{\alpha\beta} B_{mnp}$ into the first equation of (3.3), one finds that

$$(\gamma^s \gamma_{mnpqr})_\gamma{}^\alpha H_{s\alpha} = \left(-\gamma^{stu}\gamma_{mnpqr}\right)_\gamma{}^\alpha D_\alpha B_{stu}\,, \tag{5.2}$$

which implies that

$$D_\alpha B^{mnp} = \gamma^{[m}_{\alpha\beta} Z^{np]\beta} - \frac{1}{48}\left(\gamma^{[mn}\right)_\alpha{}^\beta H^{p]}_\beta + \gamma^{mnp}_{\alpha\gamma} Y^\gamma \tag{5.3}$$

for some Y^γ and $Z^{np\beta}$ satisfying $Z^{np\beta}\gamma_{p\alpha\beta} = 0$. It will now be argued that (5.3) implies that B_{mnp} describes a massive spin-two multiplet whose mass will be determined by the fifth equation of (3.3).

To analyze the physical content of (5.3), it will be useful to choose a reference frame in which the spatial momenta $k_a = 0$ for $a = 1$ to 9, where the indices $[a, b, c, \ldots]$ denote spatial directions.[1] This reference frame is always possible since the fifth equation of (3.3) at $\alpha' = 0$ implies that $\gamma^{\alpha\beta}_{mnp}(B_{\alpha\beta} + D_\alpha A_\beta) = 0$, so B_{mnp} has no massless solutions. The spatial polarizations of (5.3) imply that

$$D_\alpha B^{abc} = \gamma^{[a}_{\alpha\beta} S^{bc]\beta} \tag{5.4}$$

for some superfield $S^{bc\beta}$. To show that B^{bcd} describes a massive spin-two multiplet, recall that massive representations of $D = 10$ supersymmetry (or massless representations of $D = 11$ supersymmetry) are described by the states Ω^P_A, where P indices range over the 128 bosonic and 128 fermionic components of the smallest SO(9) supersymmetric multiplet and A indices describe the degeneracy of the "ground" state. Note that supersymmetry transformations act only on the P index and leave the A index invariant.

Defining $b^{bcd} = B^{bcd}|_{\theta=0}$, the indices $[bcd]$ on b^{bcd} could in principle come from contractions of P indices with A indices. But the constraint of (5.4) implies that the supersymmetry transformation of b^{bcd} is $\delta b^{bcd} = (\epsilon\gamma^{[b} S^{cd]})|_{\theta=0}$, which implies through the supersymmetry transformation of the P index that all indices in b^{bcd} come from P. So the "ground state" Ω^P is non-degenerate and b^{bcd} is the three-form of the smallest SO(9) supersymmetric multiplet. Furthermore, the supersymmetry transformation of b^{bcd} implies that

$$S^{bc\beta} = \left(\gamma^{[b}\Psi^{c]}\right)^\beta\,, \tag{5.5}$$

[1] Although the analysis would be more complicated, the physical content of (5.3) could also be covariantly derived by applying combinations of D_α and using γ-matrix identities.

where $\gamma_c^{\beta\gamma}\Psi_\gamma^c = 0$ and $(\Psi_\gamma^c)|_{\theta=0} = \psi_\gamma^c$ is the spin 3/2 field. The remaining 44 bosonic degrees of freedom in the SO(9) multiplet are described by the $\theta = 0$ components of the superfield $G^{bc} = D\gamma^{(b}\Psi^{c)}$ which satisfies $\eta_{bc}G^{bc} = 0$. Since G^{mn} is a spin-two superfield, one can interpret $H_{n\alpha}$ as a $D = 10$ vector-spinor prepotential as in [10]. Note that in $D = 4$, a similar role is played by a vector prepotential for a massive spin-two superfield [11, 12].

To complete the proof that B^{mnp} describes the massive spin-two multiplet of (5.1), it will now be shown that $B^{0bc} = 0$ when $k_a = 0$ for $a = 1$ to 9. Comparing (5.3), (5.4) and (5.5), one finds that

$$Y^\gamma = 0\,, \qquad Z^{bc} = h\left(\gamma^{[b}\Psi^{c]}\right)^\gamma, \qquad H_\beta^b = 96(h-1)\Psi_\beta^b \tag{5.6}$$

for some constant h. And $(\gamma_m Z^{mn})_\alpha = 0$ implies that $Z^{0b\gamma} = -7h(\gamma^0\Psi^b)^\gamma$. After using the gauge parameter Ω_2^α to gauge $(\gamma^m H_m)^\alpha = 0$, one learns from (5.3) that

$$D_\alpha B^{0bc} = (4-16h)\left(\gamma^0\gamma^{[b}\Psi^{c]}\right)_\alpha. \tag{5.7}$$

Using similar arguments as before, one can argue that the only solution to (5.7) is $B^{0bc} = 0$ and $h = 1/4$. To prove this, note that $b^{obc} = B^{0bc}|_{\theta=0}$ transforms under supersymmetry as $\delta b^{obc} = (4-16h)(\epsilon\gamma^0\gamma^{[b}\psi^{c]})$. But there are no states in Ω_A^P which transform in this manner, so b^{0bc} must vanish.

So (5.3) implies that B^{mnp} describes a massive spin-two multiplet. Furthermore, after using the gauge parameters Ω_4, Ω_{5mn} and (2.7) to gauge-fix

$$C^\alpha{}_\beta = (\gamma^{mnpq})^\alpha{}_\beta C_{mnpq} \qquad \text{and} \qquad \gamma^{m\alpha\beta}F_{\beta mn} = 0\,, \tag{5.8}$$

the first three equations of (3.3) imply that $[H_{m\alpha}, C_{mnpq}, E_\alpha, F_{\alpha mn}]$ are determined from B^{mnp} by the equations

$$\begin{aligned} H_\alpha^p &= \frac{3}{7}(\gamma_{mn}D)_\alpha B^{mnp}\,, \qquad C_{mnpq} = \frac{1}{48}\partial_{[m}B_{npq]}\,, \qquad E_\alpha = 0\,, \\ F_{\alpha mn} &= \frac{7}{16}\partial_{[m}H_{n]\alpha} - \frac{1}{16}\partial^q(\gamma_{q[m})^\beta{}_\alpha H_{n]\beta}\,, \end{aligned} \tag{5.9}$$

and the trace of the seventh equation of (3.3) implies that

$$K^s_{mnpq} = \frac{1}{1920}\left(\gamma^{\alpha\beta}_{mnpqu}D_\alpha F^{su}_\beta - \frac{1}{72}\gamma^{\alpha\beta}_{ru[mnp}\delta^s_{q]}D_\alpha F^{ru}_\beta\right). \tag{5.10}$$

Plugging (5.9) and (5.10) into the fourth equation of (3.3) implies that $\gamma^{\alpha\beta}_{mnpqr}D_\alpha A_\beta = 0$, so one can gauge-fix $A_\beta = 0$ using the remaining gauge transformation parameterized by $\Omega_{1\beta}$. And plugging (5.9) and (5.10) into the fifth equation of (3.3) implies that

$$\left(\partial_m\partial^m - \frac{1}{\alpha'}\right)B_{npq} = 0 \tag{5.11}$$

so that $(\text{mass})^2 = 1/\alpha'$. Finally, the sixth equation and the traceless part of the seventh equation of (3.3) provide no new information, as can be seen from the fact that if the first five equations of (3.3) are satisfied,

$$QV =: J(\lambda\gamma^{mnpqr}\lambda) : S_{mnpqr} + : N_{st}(\lambda\gamma^{mnpqr}\lambda) : T^{st}_{mnpqr} \tag{5.12}$$

for some S_{mnpqr} and traceless T^{st}_{mnpqr}. But $Q^2 = 0$ implies that

$$0 = Q\left[: J(\lambda\gamma^{mnqpr}\lambda) : S_{mnpqr} + : N_{st}(\lambda\gamma^{mnpqr}\lambda) : T^{st}_{mnpqr}\right]$$
$$= -\frac{\alpha'}{2}\lambda^\alpha d_\alpha(\lambda\gamma^{mnqpr}\lambda)S_{mnpqr} + \frac{\alpha'}{4}(\lambda\gamma_{st}d)(\lambda\gamma^{mnpqr}\lambda)T^{st}_{mnpqr} + \cdots, \qquad (5.13)$$

where $\cdots$ does not involve d_α. So $Q^2 = 0$ implies that $S_{mnpqr} = T^{st}_{mnpqr} = 0$.

So it has been shown that the vertex operator of (2.6) describes a spin-two multiplet with $(\text{mass})^2 = 1/\alpha'$ in terms of ten-dimensional superfields.

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